SL Paper 2

Let $u = 6i + 3j + 6k$ and $v = 2i + 2j + k$.

a. Find [5]

(i) $u \bullet v$;

(ii) $|u|$;

(iii) $|v|$.

b. Find the angle between u and v. [2]

Markscheme

a. (i) correct substitution ***(A1)***

eg $6 \times 2 + 3 \times 2 + 6 \times 1$

$u \bullet v = 24$ ***A1 N2***

(ii) correct substitution into magnitude formula for u or v ***(A1)***

eg $\sqrt{6^2 + 3^2 + 6^2},\ \sqrt{2^2 + 2^2 + 1^2}$, correct value for $|v|$

$|u| = 9$ ***A1 N2***

(iii) $|v| = 3$ ***A1 N1***

[5 marks]

b. correct substitution into angle formula ***(A1)***

eg $\frac{24}{9\times3},\ 0.\overline{8}$

$0.475882,\ 27.26604^\circ$ ***A1 N2***

$0.476,\ 27.3^\circ$

[2 marks]

Total [7 marks]

Examiners report

a. Many candidates performed well in this question. Some candidates were unfamiliar with the basis vector notation and wrongly substituted the ***i-j-k*** into the formulas. Others occasionally assumed that the magnitude could be negative.

b. Many candidates performed well in this question. Some candidates were unfamiliar with the basis vector notation and wrongly substituted the ***i-j-k*** into the formulas. Others occasionally assumed that the magnitude could be negative.

Two lines with equations $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P. Find the coordinates of P.

Markscheme

evidence of appropriate approach ***(M1)***

e.g. $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$

two correct equations ***A1A1***

e.g. $2 + 5s = 9 - 3t$, $3 - 3s = 2 + 5t$, $-1 + 2s = 2 - t$

attempting to solve the equations ***(M1)***

one correct parameter $s = 2$, $t = -1$ ***A1***

P is $(12, -3, 3)$ (accept $\begin{pmatrix} 12 \\ -3 \\ 3 \end{pmatrix}$) ***A1*** ***N3***

[6 marks]

Examiners report

If this topic had been taught well then the candidates scored highly. The question was either well answered or not at all. Many candidates did not understand what was needed and tried to find the length of vectors or mid-points of lines. The other most common mistake was to use the values of the parameters to write the coordinates as $\mathrm{P}(2,\ -1)$.

Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\frac{\pi}{3}$.

Find the value of k .

Markscheme

correct substitutions for $\boldsymbol{v} \bullet \boldsymbol{w}$; $|\boldsymbol{v}|$; $|\boldsymbol{w}|$ ***(A1)(A1)(A1)***

e.g. $2k + (-3) \times (-2) + 6 \times 4$, $2k + 30$; $\sqrt{2^2 + (-3)^2 + 6^2}$, $\sqrt{49}$; $\sqrt{k^2 + (-2)^2 + 4^2}$, $\sqrt{k^2 + 20}$

evidence of substituting into the formula for scalar product ***(M1)***

e.g. $\frac{2k+30}{7\times\sqrt{k^2+20}}$

correct substitution ***A1***

e.g. $\cos\frac{\pi}{3} = \frac{2k+30}{7\times\sqrt{k^2+20}}$

$k = 18.8$ ***A2*** ***N5***

[7 marks]

Examiners report

For the most part, this question was well done and candidates had little difficulty finding the scalar product, the appropriate magnitudes and then correctly substituting into the formula for the angle between vectors. However, few candidates were able to solve the resulting equation using their GDCs to obtain the correct answer. Problems arose when candidates attempted to solve the resulting equation analytically.

Let $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

a. Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

b. Let $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $\mathrm{B\hat{A}C}$. [4]

Markscheme

a. correct substitution ***(A1)***

eg $\sqrt{4^2+1^2+2^2}$

4.58257

$\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{21}$ (exact), 4.58 ***A1 N2***

[2 marks]

b. finding scalar product and $\left|\overrightarrow{\mathrm{AC}}\right|$ ***(A1)(A1)***

scalar product $= (4\times 3)+(1\times 0)+(2\times 0)\ (=12)$

$\left|\overrightarrow{\mathrm{AC}}\right| = \sqrt{3^2+0+0}\ (=3)$

substituting **their** values into cosine formula ***(M1)***

eg cos $\mathrm{B\hat{A}C} = \frac{4\times3+0+0}{\sqrt{3^2}\times\sqrt{21}}$, $\frac{4}{\sqrt{21}}$, $\cos\theta = 0.873$

0.509739 (29.2059°)

$\mathrm{B\hat{A}C} = 0.510$ (29.2°) ***A1 N2***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

a(i) and (ii) (i) Write down the coordinates of A. [4]

(ii) Find the speed of the airplane in ms^{-1}.

b(i) and (ii) After seven seconds the airplane passes through a point B. [5]

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.

c. Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}$, $a \in \mathbb{R}$. [7]

The angle between the flight paths of Airplane 1 and Airplane 2 is 40°. Find the two values of a.

Markscheme

a(i) and (ii) (i) $(3, -4, 0)$ ***A1 N1***

(ii) choosing velocity vector $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ ***(M1)***

finding magnitude of velocity vector ***(A1)***

e.g. $\sqrt{(-2)^2 + 3^2 + 1^2}$, $\sqrt{4+9+1}$

speed $= 3.74$ $\left(\sqrt{14}\right)$ ***A1 N2***

[4 marks]

b(i) and (ii) (i) substituting $p = 7$ ***(M1)***

$\text{B} = (-11, 17, 7)$ ***A1 N2***

(ii) **METHOD 1**

appropriate method to find $\overrightarrow{\text{AB}}$ or $\overrightarrow{\text{BA}}$ ***(M1)***

e.g. $\overrightarrow{\text{AO}} + \overrightarrow{\text{OB}}$, $\text{A} - \text{B}$

$\overrightarrow{\text{AB}} = \begin{pmatrix} -14 \\ 21 \\ 7 \end{pmatrix}$ or $\overrightarrow{\text{BA}} = \begin{pmatrix} 14 \\ -21 \\ -7 \end{pmatrix}$ ***(A1)***

distance $= 26.2$ $\left(7\sqrt{14}\right)$ ***A1 N3***

METHOD 2

evidence of applying distance is speed × time ***(M2)***

e.g. 3.74×7

distance $= 26.2$ $\left(7\sqrt{14}\right)$ ***A1 N3***

METHOD 3

attempt to find AB^2, AB ***(M1)*** e.g. $(3-(-11))^2+(-4-17)^2+(0-7))^2$, $\sqrt{(3-(-11))^2+(-4-17)^2+(0-7))^2}$

$\text{AB}^2=686$, $\text{AB}=\sqrt{686}$ ***(A1)***

distance $\text{AB}=26.2$ $\left(7\sqrt{14}\right)$ ***A1*** ***N3***

[5 marks]

c. correct direction vectors $\begin{pmatrix}-2\\3\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\a\end{pmatrix}$ ***(A1)(A1)***

$\left|\begin{matrix}-1\\2\\a\end{matrix}\right|=\sqrt{a^2+5}$, $\begin{pmatrix}-2\\3\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\a\end{pmatrix}=a+8$ ***(A1)(A1)***

substituting ***M1***

e.g. $\cos 40^\circ=\frac{a+8}{\sqrt{14}\sqrt{a^2+5}}$

$a=3.21$, $a=-0.990$ ***A1A1*** ***N3***

[7 marks]

Examiners report

a(i) and (ii). Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

b(i) and (ii). Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

Some knew that speed and distance were magnitudes of vectors but chose the wrong vectors to calculate magnitudes.

c. Very few candidates were able to get the two correct answers in (c) even if they set up the equation correctly. Much contorted algebra was seen and little evidence of using the GDC to solve the equation. Many made simple algebraic errors by combining unlike terms in working with the scalar product (often writing $8a$ rather than $8+a$) or the magnitude (often writing $5a^2$ rather than $5+a^2$).

Consider the points P(2, −1, 5) and Q(3, − 3, 8). Let L_1 be the line through P and Q.

a. Show that $\overrightarrow{\text{PQ}}=\begin{pmatrix}1\\-2\\3\end{pmatrix}$. [1]

b. The line L_1 may be represented by $\boldsymbol{r}=\begin{pmatrix}3\\-3\\8\end{pmatrix}+s\begin{pmatrix}1\\-2\\3\end{pmatrix}$. [3]

(i) What information does the vector $\begin{pmatrix}3\\-3\\8\end{pmatrix}$ give about L_1 ?

(ii) Write down another vector representation for L_1 using $\begin{pmatrix}3\\-3\\8\end{pmatrix}$.

c. The point $\mathrm{T}(-1, 5, p)$ lies on L_1 . [3]

Find the value of p .

d. The point T also lies on L_2 with equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 9 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ q \end{pmatrix}$. [3]

Show that $q = -3$.

e. Let θ be the **obtuse** angle between L_1 and L_2 . Calculate the size of θ . [7]

Markscheme

a. evidence of correct approach ***A1***

e.g. $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{OQ}} - \overrightarrow{\mathrm{OP}}$, $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ ***AG N0***

[1 mark]

b. (i) correct description ***R1 N1***

e.g. reference to $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$ being the position vector of a point on the line, a vector to the line, a point on the line.

(ii) **any** correct expression in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2 N2***

where $\boldsymbol{a}$ is $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

e.g. $\boldsymbol{r} = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} 3+2s \\ -3-4s \\ 8+6s \end{pmatrix}$

[3 marks]

c. **one** correct equation ***(A1)***

e.g. $3 + s = -1$, $-3 - 2s = 5$

$s = -4$ ***A1***

$p = -4$ ***A1 N2***

[3 marks]

d. one correct equation ***A1***

e.g. $-3 + t = -1$, $9 - 2t = 5$

$t = 2$ ***A1***

substituting $t = 2$

e.g. $2 + 2q = -4$, $2q = -6$ ***A1***

$q = -3$ ***AG N0***

[3 marks]

e. choosing correct direction vectors $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$ ***(A1)(A1)***

finding correct scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $(1)(1) + (-2)(-2) + (-3)(3) \quad (= -4)$

magnitudes $\sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$, $\sqrt{1^2 + (-2)^2 + (-3)^2} = \sqrt{14}$

evidence of substituting into scalar product ***M1***

e.g. $\cos\theta = \frac{-4}{3.741\ldots \times 3.741\ldots}$

$\theta = 1.86$ radians (or 107°) ***A1*** ***N4***

[7 marks]

Examiners report

a. Most candidates answered part (a) easily.

b. For part (b), a number of candidates stated that the vector was a "starting point," which misses the idea that it is a position vector to some point on the line.

c. Parts (c) and (d) proved accessible to many.

d. Parts (c) and (d) proved accessible to many.

e. For part (e), a surprising number of candidates chose incorrect vectors. Few candidates seemed to have a good conceptual understanding of the vector equation of a line.

Consider the points $\mathrm{A}\ (1,\ 5,\ -7)$ and $\mathrm{B}\ (-9,\ 9,\ -6)$.

Let C be a point such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

The line L passes through B and is parallel to (AC).

a. Find $\overrightarrow{\mathrm{AB}}$. [2]

b. Find the coordinates of C. [2]

c. Write down a vector equation for L. [2]

d. Given that $\left|\overrightarrow{\mathrm{AB}}\right| = k\left|\overrightarrow{\mathrm{AC}}\right|$, find k. [3]

e. The point D lies on L such that $\left|\overrightarrow{\mathrm{AB}}\right| = \left|\overrightarrow{\mathrm{BD}}\right|$. Find the possible coordinates of D. [6]

Markscheme

a. valid approach ***(M1)***

eg $\mathrm{B}-\mathrm{A},\ \mathrm{AO}+\mathrm{OB},\ \begin{pmatrix}-9\\9\\-6\end{pmatrix}-\begin{pmatrix}1\\5\\-7\end{pmatrix}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}-10\\4\\1\end{pmatrix}$ ***A1 N2***

[2 marks]

b. valid approach ***(M1)***

eg $\mathrm{OC}=\mathrm{OA}+\mathrm{AC},\ \begin{pmatrix}1+6\\5-4\\-7+0\end{pmatrix}$

$\mathrm{C}(7,\ 1,\ -7)$ ***A1 N2***

[2 marks]

c. any correct equation in the form ***r*** $=$ ***a*** $+t$***b*** (accept any parameter for t

where ***a*** is $\begin{pmatrix}-9\\9\\-6\end{pmatrix}$, and ***b*** is a scalar multiple of $\begin{pmatrix}6\\-4\\0\end{pmatrix}$ ***A2 N2***

eg ***r*** $=\begin{pmatrix}-9\\9\\-6\end{pmatrix}+t\begin{pmatrix}6\\-4\\0\end{pmatrix}$, ***r*** $=-9$***i*** $+9$***j*** -6***k*** $+s(6$***i*** -4***j*** $+0$***k***$)$

[2 marks]

d. correct magnitudes ***(A1)(A1)***

eg $\sqrt{(-10)^2+(-4)^2+1^2},\ \sqrt{6^2+(-4)^2+(0)^2},\ \sqrt{10^2+4^2+1},\ \sqrt{6^2+4^2}$

$k=\frac{\sqrt{117}}{\sqrt{52}}\ (=1.5)\ (\text{exact})$ ***A1 N3***

[3 marks]

e. correct interpretation of relationship between magnitudes ***(A1)***

eg $\mathrm{AB}=1.5\mathrm{AC},\ \mathrm{BD}=1.5\mathrm{AC},\ \sqrt{117}=\sqrt{52t^2}$

recognizing D can have two positions (may be seen in working) ***R1***

eg $\overrightarrow{\mathrm{BD}}=1.5\overrightarrow{\mathrm{AC}}$ and $\overrightarrow{\mathrm{BD}}=-1.5\overrightarrow{\mathrm{AC}},\ t=\pm1.5$, diagram, two answers

valid approach (seen anywhere) ***(M1)***

eg $\overrightarrow{\mathrm{OD}}=\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{BD}},\ \begin{pmatrix}-9\\9\\-6\end{pmatrix}+t\begin{pmatrix}6\\-4\\0\end{pmatrix},\ \overrightarrow{\mathrm{BD}}=k\begin{pmatrix}6\\-4\\0\end{pmatrix}$

one correct expression for $\overrightarrow{\mathrm{OD}}$ ***(A1)***

eg $\overrightarrow{\mathrm{OD}}=\begin{pmatrix}-9\\9\\-6\end{pmatrix}+1.5\begin{pmatrix}6\\-4\\0\end{pmatrix},\ \begin{pmatrix}-9\\9\\-6\end{pmatrix}-1.5\begin{pmatrix}6\\-4\\0\end{pmatrix}$

$\mathrm{D}=(0,\ 3,\ -6),\ \mathrm{D}=(-18,\ 15,\ -6)$ (accept position vectors) ***A1A1 N3***

[6 marks]

Examiners report

a. Parts (a) and (b) were attempted by the great majority of the candidates and appropriate approaches were seen, earning at least the method marks.

b. Parts (a) and (b) were attempted by the great majority of the candidates and appropriate approaches were seen, earning at least the method marks.

c. Part (c) was generally well done, with many candidates writing the equation of the line as $L = \boldsymbol{a} + t\boldsymbol{b}$, losing one mark.

d. Part (d) was also well answered by a great majority of students. Even those candidates who had part (a) incorrect, could gain all the marks here.

e. Part (e) was the most challenging of the paper. For many a major problem was to set up the equation $\sqrt{117} = \sqrt{52t^2}$ and, hence, realize that D could have two positions.

The point O has coordinates (0 , 0 , 0) , point A has coordinates (1 , – 2 , 3) and point B has coordinates (– 3 , 4 , 2) .

a(i) and (ii). [8]

(i) Show that $\overrightarrow{\text{AB}} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $\text{B}\hat{\text{A}}\text{O}$.

b. [2]

The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1 .

c(i) and (ii). [6]

The line L_2 passes through A and is parallel to $\overrightarrow{\text{OB}}$.

(i) Find a vector equation for L_2 , giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Point $C(k, -k, 5)$ is on L_2 . Find the coordinates of C.

d. [2]

The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ and passes through the point C.

Find the value of p at C.

Markscheme

a(i) and (ii).

(i) evidence of approach ***M1***

e.g. $\overrightarrow{\text{AO}} + \overrightarrow{\text{OB}} = \overrightarrow{\text{AB}}$

$\overrightarrow{\text{AB}} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$ ***AG N0***

(ii) for choosing **correct** vectors, ($\overrightarrow{\text{AO}}$ with $\overrightarrow{\text{AB}}$ or $\overrightarrow{\text{OA}}$ with $\overrightarrow{\text{BA}}$) ***(A1)(A1)***

Note: Using $\overrightarrow{\text{AO}}$ with $\overrightarrow{\text{BA}}$ will lead to $\pi - 0.799$. If they then say $\text{B}\hat{\text{A}}\text{O} = 0.799$, this is a correct solution.

calculating $\overrightarrow{\text{AO}} \bullet \overrightarrow{\text{AB}}$, $\left|\overrightarrow{\text{AO}}\right|$, $\left|\overrightarrow{\text{AB}}\right|$ ***(A1)(A1)(A1)*** e.g. $d_1 \bullet d_2 = (-1)(-4) + (2)(6) + (-3)(-1) (= 19)$

$|d_1| = \sqrt{(-1)^2 + 2^2 + (-3)^2} (= \sqrt{14})$, $|d_2| = \sqrt{(-4)^2 + 6^2 + (-1)^2} (= \sqrt{53})$

evidence of using the formula to find the angle ***M1***

e.g. $\cos\theta = \frac{(-1)(-4)+(2)(6)+(-3)(-1)}{\sqrt{(-1)^2+2^2+(-3)^2}\sqrt{(-4)^2+6^2+(-1)^2}}$, $\frac{19}{\sqrt{14}\sqrt{53}}$, $0.69751\ldots$

$\mathrm{B\widehat{A}O} = 0.799$ radians (accept $45.8°$) ***A1*** ***N3***

[8 marks]

b. two correct answers ***A1A1***

e.g. $(1, -2, 3)$, $(-3, 4, 2)$, $(-7, 10, 1)$, $(-11, 16, 0)$ ***N2***

[2 marks]

c(i) and (ii).
(i) $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ ***A2*** ***N2***

(ii) C on L_2 , so $\begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ ***(M1)***

evidence of equating components ***(A1)***

e.g. $1 - 3t = k$, $-2 + 4t = -k$, $5 = 3 + 2t$

one correct value $t = 1$, $k = -2$ (seen anywhere) ***(A1)***

coordinates of C are $(-2, 2, 5)$ ***A1*** ***N3***

[6 marks]

d.
for setting up one (or more) correct equation using $\begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ ***(M1)***

e.g. $3 + p = -2$, $-8 - 2p = 2$, $-p = 5$

$p = -5$ ***A1*** ***N2***

[2 marks]

Examiners report

a(i) and (ii). Part (ai) was done well by most students. Most knew how to approach finding the angle in part (aii). The problems occurred when the incorrect vectors were chosen. If the vectors being used were stated, then follow through marks could be given.

b. Part (b) was well done.

c(i) and (ii). In part (ci), the error that occurred most often was the incorrect choice for the direction vector.

d. Those that were able to find the coordinates in part (cii) were also able to be successful in part (d).

Consider the lines L_1 and L_2 with equations $L_1 : \boldsymbol{r} = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and $L_2 : \boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P.

a. Find the coordinates of P. [6]

b. Show that the lines are perpendicular. [5]

c. The point $Q(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2. [6]

Find the coordinates of R.

Markscheme

a. appropriate approach ***(M1)***

eg $\begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$, $L_1 = L_2$

any **two** correct equations ***A1A1***

eg $11 + 4s = 1 + 2t,\ 8 + 3s = 1 + t,\ 2 - s = -7 + 11t$

attempt to solve system of equations ***(M1)***

eg $10 + 4s = 2(7 + 3s), \begin{cases} 4s - 2t = -10 \\ 3s - t = -7 \end{cases}$

one correct parameter ***A1***

eg $s = -2,\ t = 1$

$P(3, 2, 4)$ (accept position vector) ***A1*** ***N3***

[6 marks]

b. choosing correct direction vectors for L_1 and L_2 ***(A1)(A1)***

eg $\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$ (or any scalar multiple)

evidence of scalar product (with any vectors) ***(M1)***

eg $a \cdot b, \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$

correct substitution ***A1***

eg $4(2) + 3(1) + (-1)(11),\ 8 + 3 - 11$

calculating $a \cdot b = 0$ ***A1***

Note: Do not award the final ***A1*** without evidence of calculation.

vectors are perpendicular ***AG*** ***N0***

[5 marks]

c. **Note:** Candidates may take different approaches, which do not necessarily involve vectors.

In particular, most of the working could be done on a diagram. Award marks in line with the markscheme.

METHOD 1

attempt to find $\overrightarrow{QP}$ or $\overrightarrow{PQ}$ ***(M1)***

correct working (may be seen on diagram) ***A1***

eg $\overrightarrow{QP} = \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix}, \overrightarrow{PQ} = \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg $\overrightarrow{\mathrm{QP}} = \overrightarrow{\mathrm{PR}}$, marked on diagram

correct working ***(A1)***

eg $\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

$\mathrm{R}(-1, -1, 5)$ (accept position vector) ***A1*** ***N3***

METHOD 2

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg P midpoint of QR, marked on diagram

valid approach to find **one** coordinate of mid-point ***(M1)***

eg $x_p = \frac{x_Q + x_R}{2},\ 2y_p = y_Q + y_R,\ \frac{1}{2}(z_Q + z_R)$

one correct substitution ***A1***

eg $x_R = 3 + (3 - 7),\ 2 = \frac{5 + y_R}{2},\ 4 = \frac{1}{2}(z + 3)$

correct working for one coordinate ***(A1)***

eg $x_R = 3 - 4,\ 4 - 5 = y_R,\ 8 = (z + 3)$

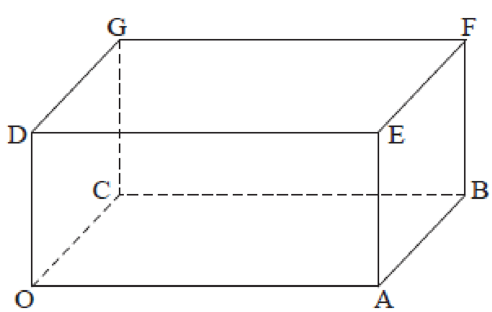
$\mathrm{R}(-1, -1, 5)$ (accept position vector) ***A1*** ***N3***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and $\overrightarrow{\mathrm{OA}} = 4\boldsymbol{i}$, $\overrightarrow{\mathrm{OC}} = 3\boldsymbol{j}$, $\overrightarrow{\mathrm{OD}} = 2\boldsymbol{k}$.



a(i), (ii) and (iii). (i) Find $\overrightarrow{\mathrm{OB}}$. [5]

(ii) Find $\overrightarrow{\mathrm{OF}}$.

(iii) Show that $\overrightarrow{\mathrm{AG}} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$.

b(i) and (ii). Write down a vector equation for [4]

(i) the line OF;

(ii) the line AG.

c. Find the obtuse angle between the lines OF and AG. [7]

Markscheme

a(i), (ii) and (iii). (i) valid approach *(M1)*

e.g. $\mathrm{OA}+\mathrm{OB}$

$\overrightarrow{\mathrm{OB}}=4\boldsymbol{i}+3\boldsymbol{j}$ *A1 N2*

(ii) valid approach *(M1)*

e.g. $\overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BF}}$; $\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{BF}}$; $\overrightarrow{\mathrm{OC}}+\overrightarrow{\mathrm{CG}}+\overrightarrow{\mathrm{GF}}$

$\overrightarrow{\mathrm{OF}}=4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k}$ *A1 N2*

(iii) correct approach *A1*

e.g. $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OC}}+\overrightarrow{\mathrm{CG}}$; $\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BF}}+\overrightarrow{\mathrm{FG}}$; $\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}+\overrightarrow{\mathrm{CG}}$

$\overrightarrow{\mathrm{AG}}=-4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k}$ *AG N0*

[5 marks]

b(i) and (ii). (i) any correct equation for (OF) in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ *A2 N2*

where $\boldsymbol{a}$ is 0 or $4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k}$, and $\boldsymbol{b}$ is a scalar multiple of $4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k}$

e.g. $\boldsymbol{r}=t(4, 3, 2)$, $\boldsymbol{r}=\begin{pmatrix}4t\\3t\\2t\end{pmatrix}$, $\boldsymbol{r}=4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k}+t(4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k})$

(ii) any correct equation for (AG) in the form $\boldsymbol{r}=\boldsymbol{a}+s\boldsymbol{b}$ *A2 N2*

where $\boldsymbol{a}$ is $4\boldsymbol{i}$ or $3\boldsymbol{j}+2\boldsymbol{k}$ and $\boldsymbol{b}$ is a scalar multiple of $-4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k}$

e.g. $\boldsymbol{r}=(4, 0, 0)+s(-4, 3, 2)$, $\boldsymbol{r}=\begin{pmatrix}4-4s\\3s\\2s\end{pmatrix}$, $\boldsymbol{r}=3\boldsymbol{j}+2\boldsymbol{k}+s(-4\boldsymbol{i}+3\boldsymbol{j}+2\boldsymbol{k})$

[4 marks]

c. choosing correct direction vectors, $\overrightarrow{\mathrm{OF}}$ and $\overrightarrow{\mathrm{AG}}$ *(A1)(A1)*

scalar product $=-16+9+4\ (=-3)$ *(A1)*

magnitudes $\sqrt{4^2+3^2+2^2}$, $\sqrt{(-4)^2+3^2+2^2}$, $\left(\sqrt{29}, \sqrt{29}\right)$ *(A1)(A1)*

substitution into formula *M1*

e.g. $\cos\theta=\frac{-16+9+4}{\left(\sqrt{4^2+3^2+2^2}\right)\times\sqrt{(-4)^2+3^2+2^2}}=\left(-\frac{3}{29}\right)$

95.93777° , 1.67443 radians

$\theta=95.9^\circ$ or 1.67 *A1 N4*

[7 marks]

Examiners report

a(i), (ii) and (iii). Although a large proportion of candidates managed to answer this question, their biggest challenge was the use of a proper notation to represent the vectors and vector equations of lines.

In part (a), finding $\overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OF}}$ was generally well done, although many lost the mark for (iii) due to poor working or not clearly showing the result.

b(i) and (ii). Part (b) was very poorly done. Not all the students recognized which correct position vectors they had to use to write the equations of the lines.

It was seen that they frequently failed to present the equations in the required format, which prevented these candidates from achieving full marks. The notations generally seen were $\mathrm{AG} = \boldsymbol{a} + \boldsymbol{b}t$, $\boldsymbol{r} = 4 + t(4,\ 3,\ 2)$ or $L = \boldsymbol{a} + \boldsymbol{b}t$.

c. Most achieved the correct result in part (c) with many others gaining most of the marks as follow through from choosing incorrect vectors. Some students did not state which vectors had been used, another cause for losing marks. A few showed poor notation, including $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ in the working.

Consider the points $\mathrm{A}(5,\ 2,\ 1)$, $\mathrm{B}(6,\ 5,\ 3)$, and $\mathrm{C}(7,\ 6,\ a + 1)$, $a \in \mathbb{R}$.

Let q be the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$.

a. Find [3]

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$.

b. Find the value of a for which $\mathrm{q} = \frac{\pi}{2}$. [4]

c. i. Show that $\cos q = \frac{2a+14}{\sqrt{14a^2+280}}$. [8]

ii. Hence, find the value of a for which $\mathrm{q} = 1.2$.

c.ii. Hence, find the value of a for which $\mathrm{q} = 1.2$. [4]

Markscheme

a. (i) appropriate approach ***(M1)***

eg $\overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OB}}$, $\mathrm{B} - \mathrm{A}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ ***A1 N2***

(ii) $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$ ***A1 N1***

[3 marks]

b. valid reasoning (seen anywhere) ***R1*** *eg* scalar product is zero, $\cos\frac{\pi}{2}=\frac{\boldsymbol{u}\bullet\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$

correct scalar product of **their** $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ (may be seen in part (c)) ***(A1)***

eg $1(2)+3(4)+2(a)$

correct working for **their** $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ ***(A1)***

eg $2a+14$, $2a=-14$

$a=-7$ ***A1 N3***

[4 marks]

c. correct magnitudes (may be seen in (b)) ***(A1)(A1)***

$\sqrt{1^2+3^2+2^2}\left(=\sqrt{14}\right)$, $\sqrt{2^2+4^2+a^2}\left(=\sqrt{20+a^2}\right)$

substitution into formula ***(M1)***

eg $\cos\theta\frac{1\times2+3\times4+2\times a}{\sqrt{1^2+3^2+2^2}\sqrt{2^2+4^2+a^2}}$, $\frac{14+2a}{\sqrt{14}\sqrt{4+16+a^2}}$

simplification leading to required answer ***A1***

eg $\cos\theta=\frac{14+2a}{\sqrt{14}\sqrt{20+a^2}}$

$\cos\theta=\frac{2a+14}{\sqrt{14a^2+280}}$ ***AG N0***

[4 marks]

correct setup ***(A1)***

eg $\cos1.2=\frac{2a+14}{\sqrt{14a^2+280}}$

valid attempt to solve ***(M1)***

eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos1.2=0$, attempt to square

$a=-3.25$ ***A2 N3***

[4 marks]

c.ii. correct setup ***(A1)***

eg $\cos1.2=\frac{2a+14}{\sqrt{14a^2+280}}$

valid attempt to solve ***(M1)***

eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos1.2=0$, attempt to square

$a=-3.25$ ***A2 N3***

[4 marks]

Examiners report

a. The majority of candidates successfully found the vectors between the given points in part (a).

b. In part (b), while most candidates correctly found the value of a, many unnecessarily worked with the magnitudes of the vectors, sometimes leading to algebra errors.

c. Some candidates showed a minimum of working in part (c)(i); in a “show that” question, candidates need to ensure that their working clearly leads to the answer given. A common error was simplifying the magnitude of vector AC to $\sqrt{20a^2}$ instead of $\sqrt{20+a^2}$.

In part (c)(ii), a disappointing number of candidates embarked on a usually fruitless quest for an algebraic solution rather than simply solving the resulting equation with their GDC. Many of these candidates showed quite weak algebra manipulation skills, with errors involving the square root occurring in a myriad of ways.

c.ii.In part (c)(ii), a disappointing number of candidates embarked on a usually fruitless quest for an algebraic solution rather than simply solving the resulting equation with their GDC. Many of these candidates showed quite weak algebra manipulation skills, with errors involving the square root occurring in a myriad of ways.

Let $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. The vector $\boldsymbol{v} + p\boldsymbol{w}$ is perpendicular to $\boldsymbol{w}$. Find the value of p.

Markscheme

$p\boldsymbol{w} = p\boldsymbol{i} + 2p\boldsymbol{j} - 3p\boldsymbol{k}$ (seen anywhere) ***(A1)***

attempt to find $\boldsymbol{v} + p\boldsymbol{w}$ ***(M1)***

e.g. $3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k} + p(\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})$

collecting terms $(3+p)\boldsymbol{i} + (4+2p)\boldsymbol{j} + (1-3p)\boldsymbol{k}$ ***A1***

attempt to find the dot product ***(M1)***

e.g. $1(3+p) + 2(4+2p) - 3(1-3p)$

setting **their** dot product equal to 0 ***(M1)***

e.g. $1(3+p) + 2(4+2p) - 3(1-3p) = 0$

simplifying ***A1***

e.g. $3 + p + 8 + 4p - 3 + 9p = 0$, $14p + 8 = 0$

$p = -0.571 \left(-\frac{8}{14}\right)$ ***A1*** ***N3***

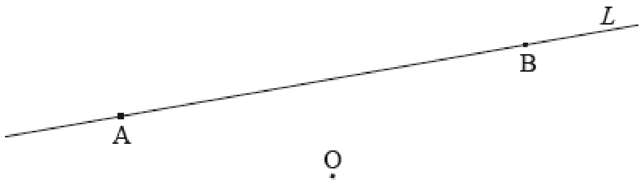
[7 marks]

Examiners report

This question was very poorly done with many leaving it blank. Of those that did attempt it, most were able to find $\boldsymbol{v} + p\boldsymbol{w}$ but really did not know how to proceed from there. They tried many approaches, such as, finding magnitudes, using negative reciprocals, or calculating the angle between two vectors. A few had the idea that the scalar product should equal zero but had trouble trying to set it up.

The points A and B lie on a line L, and have position vectors $\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$ respectively. Let O be the origin. This is shown on the following diagram.

diagram not to scale

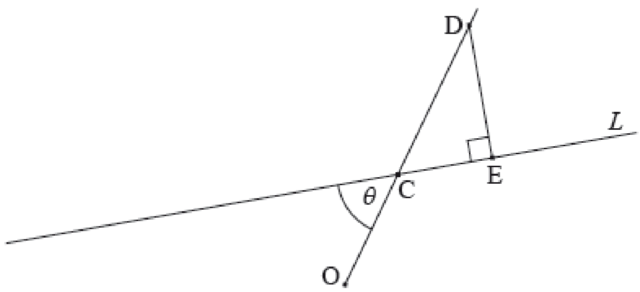


The point C also lies on L, such that $\overrightarrow{\mathrm{AC}} = 2\overrightarrow{\mathrm{CB}}$.

Let θ be the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{OC}}$.

Let D be a point such that $\overrightarrow{\mathrm{OD}} = k\overrightarrow{\mathrm{OC}}$, where $k > 1$. Let E be a point on L such that $\mathrm{C\hat{E}D}$ is a right angle. This is shown on the following diagram.

diagram not to scale



a. Find $\overrightarrow{\mathrm{AB}}$. [2]

b. Show that $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$. [[N/A

c. Find θ. [5]

d. (i) Show that $\left|\overrightarrow{\mathrm{DE}}\right| = (k-1)\left|\overrightarrow{\mathrm{OC}}\right| \sin\theta$. [6]

(ii) The distance from D to line L is less than 3 units. Find the possible values of k.

Markscheme

a. valid approach (addition or subtraction) ***(M1)***

eg $\mathrm{AO} + \mathrm{OB},\ \mathrm{B} - \mathrm{A}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$ ***A1 N2***

[2 marks]

b. **METHOD 1**

valid approach using $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ***(M1)***

eg $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix}$, $\overrightarrow{\mathrm{CB}} = \begin{pmatrix} 6-x \\ 4-y \\ -1-z \end{pmatrix}$

correct working ***A1***

eg $\begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix} = \begin{pmatrix} 12-2x \\ 8-2y \\ -2-2z \end{pmatrix}$

all three equations ***A1***

eg $x+3=12-2x,\ y+2=8-2y,\ z-2=-2-2z,$

$\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ ***AG*** ***N0***

METHOD 2

valid approach ***(M1)***

eg $\overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OA}} = 2\left(\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OC}}\right)$

correct working ***A1***

eg $3\overrightarrow{\mathrm{OC}} = 2\overrightarrow{\mathrm{OB}} + \overrightarrow{\mathrm{OA}}$

correct substitution of $\overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OA}}$ ***A1***

eg $3\overrightarrow{\mathrm{OC}} = 2\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix},\ 3\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$

$\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ ***AG*** ***N0***

METHOD 3

valid approach ***(M1)***

eg $\overrightarrow{\mathrm{AC}} = \frac{2}{3}\overrightarrow{\mathrm{AB}}$, diagram, $\overrightarrow{\mathrm{CB}} = \frac{1}{3}\overrightarrow{\mathrm{AB}}$

correct working ***A1***

eg $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix},\ \overrightarrow{\mathrm{CB}} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

correct working involving $\overrightarrow{\mathrm{OC}}$ ***A1***

eg $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix},\ \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

$\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ ***AG*** ***N0***

[3 marks]

c. finding scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $= (9\times 3) + (6\times 2) + (-3\times 0)\ (=39)$

magnitudes $\sqrt{81+36+9}\ (=11.22),\ \sqrt{9+4}\ (=3.605)$

substitution into formula ***M1***

eg $\cos\theta = \frac{(9\times 3)+12}{\sqrt{126}\times\sqrt{13}}$

$\theta = 0.270549$ (accept $15.50135°$)

$\theta = 0.271$ (accept $15.5°$) ***A1*** ***N4***

[5 marks]

d. (i) attempt to use a trig ratio ***M1***

eg $\sin\theta = \frac{\text{DE}}{\text{CD}},\ \left|\overrightarrow{\text{CE}}\right| = \left|\overrightarrow{\text{CD}}\right|\cos\theta$

attempt to express $\overrightarrow{\text{CD}}$ in terms of $\overrightarrow{\text{OC}}$ ***M1***

eg $\overrightarrow{\text{OC}} + \overrightarrow{\text{CD}} = \overrightarrow{\text{OD}},\ \text{OC} + \text{CD} = \text{OD}$

correct working ***A1***

eg $\left|k\overrightarrow{\text{OC}} - \overrightarrow{\text{OC}}\right|\sin\theta$

$\left|\overrightarrow{\text{DE}}\right| = (k-1)\left|\overrightarrow{\text{OC}}\right|\sin\theta$ ***AG*** ***N0***

(ii) valid approach involving the segment DE ***(M1)***

eg recognizing $\left|\overrightarrow{\text{DE}}\right| < 3,\ \text{DE} = 3$

correct working (accept equation) ***(A1)***

eg $(k-1)(\sqrt{13})\sin 0.271 < 3,\ k - 1 = 3.11324$

$1 < k < 4.11$ (accept $k < 4.11$ but not $k = 4.11$) ***A1*** ***N2***

[6 marks]

Examiners report

a. The majority of candidates had little difficulty with parts (a) and (c). The most common error in both these parts were unforced arithmetic errors and occasional misreads of the vectors.

b. In part (b), candidates who were successful used a variety of different approaches, and it was pleasing to see the vast majority of these being well reasoned, however, there were numerous unsuccessful responses including those who attempted to use the given vector to work backwards. A lack of appropriate vector notation often meant that ideas were not always clearly communicated.

c. The majority of candidates had little difficulty with parts (a) and (c). The most common error in both these parts were unforced arithmetic errors and occasional misreads of the vectors.

d. The majority of candidates struggled to make any progress in (d), with very few realizing that simple right-angled trigonometry could be used. Few were able to successfully express CD in terms of OC which was required to show the given result. Very few candidates attempted (d)(ii), with many unable to make the connection with results found in previous parts of the question.

Line L_1 passes through points $\text{A}(1,\ -1, 4)$ and $\text{B}(2,\ -2, 5)$.

Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

a. Find $\overrightarrow{AB}$. [2]

b. Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. [2]

c. Find the angle between L_1 and L_2. [7]

d. The lines L_1 and L_2 intersect at point C. Find the coordinates of C. [6]

Markscheme

a. appropriate approach ***(M1)***

e.g. $\overrightarrow{AO} + \overrightarrow{OB}$, $B - A$

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ ***A1 N2***

[2 marks]

b. any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2 N2***

where $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

e.g. $\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} 2+t \\ -2-t \\ 5+t \end{pmatrix}$, $\boldsymbol{r} = 2\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k} + t(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$

[2 marks]

c. choosing correct direction vectors $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ ***(A1)(A1)***

finding scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $= 1 \times 2 + -1 \times 1 + 1 \times 3$ $(= 4)$

magnitudes $\sqrt{1^2 + (-1)^2 + 1^2}$ $(= 1.73\ldots)$, $\sqrt{4+1+9}$ $(= 3.74\ldots)$

substitution into $\frac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$ (accept $\theta = \frac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$, but not $\sin\theta = \frac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$) ***M1***

e.g. $\cos\theta = \frac{1 \times 2 + -1 \times 1 + 1 \times 3}{\sqrt{1^2 + (-1)^2 + 1^2}\sqrt{2^2 + 1^2 + 3^2}}$, $\cos\theta = \frac{4}{\sqrt{42}}$

$\theta = 0.906$ $(51.9°)$ ***A1 N5***

[7 marks]

d. **METHOD 1** (from $\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

appropriate approach ***(M1)***

e.g. $\boldsymbol{p} = \boldsymbol{r}$, $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} t = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} s$

two **correct** equations ***A1A1***

e.g. $1 + t = 2 + 2s$, $-1 - t = 4 + s$, $4 + t = 7 + 3s$

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = -3$, $s = -2$

C is $(-2, 2, 1)$ ***A1*** ***N3***

METHOD 2 (from $\boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

appropriate approach ***(M1)***

e.g. $\boldsymbol{p} = \boldsymbol{r}$, $\begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} s$

two **correct** equations ***A1A1***

e.g. $2 + t = 2 + 2s$, $-2 - t = 4 + s$, $5 + t = 7 + 3s$

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = -4$, $s = -2$

C is $(-2, 2, 1)$ ***A1*** ***N3***

[6 marks]

Examiners report

a. Finding $\overrightarrow{\text{AB}}$ was generally well done, although some candidates reversed the subtraction.

b. In part (b) not all the candidates recognized that $\overrightarrow{\text{AB}}$ was the direction vector of the line, as some used the position vector of point B as the direction vector.

c. Many candidates successfully used scalar product and magnitudes in part (c), although a large number did choose vectors other than the direction vectors and many did not state clearly which vectors they were using.

d. Candidates who were comfortable on the first three parts often had little difficulty with the final part. While the resulting systems were easily solved algebraically, a surprising number of candidates did not check their solutions either manually or with technology. An occasionally seen error in the final part was using a midpoint to find C. Some candidates found the point of intersection in part (c) rather than in part (d), indicating a familiarity with the type of question but a lack of understanding of the concepts involved.

The diagram shows a parallelogram ABCD.

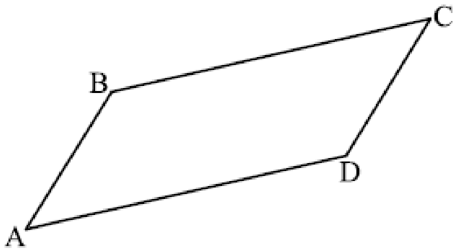


diagram not to scale

The coordinates of A, B and D are A(1, 2, 3) , B(6, 4,4) and D(2, 5, 5) .

a(i), (ii) and (iii). [5]

(i) Show that $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$.

(ii) Find $\overrightarrow{AD}$.

(iii) **Hence** show that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$.

b. Find the coordinates of point C. [3]

c(i) and (ii). [7]

(i) Find $\overrightarrow{AB} \bullet \overrightarrow{AD}$.

(ii) **Hence** find angle A.

d. Hence, or otherwise, find the area of the parallelogram. [3]

Markscheme

a(i), (ii) and (iii).

(i) evidence of approach ***M1***

e.g. $B - A$, $\overrightarrow{AO} + \overrightarrow{OB}$, $\begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ ***AG*** ***N0***

(ii) evidence of approach ***(M1)***

e.g. $D - A$, $\overrightarrow{AO} + \overrightarrow{OD}$, $\begin{pmatrix} 2 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{AD} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ ***A1*** ***N2***

(iii) evidence of approach ***(M1)***

e.g. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$

correct substitution ***A1***

e.g. $\overrightarrow{AC} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$ ***AG*** ***N0***

[5 marks]

b. evidence of combining vectors (there are at least 5 ways) ***(M1)***

e.g. $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$, $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{AD}$, $\overrightarrow{AB} = \overrightarrow{OC} - \overrightarrow{OD}$

correct substitution ***A1***

$\overrightarrow{OC} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} \left(= \begin{pmatrix} 7 \\ 7 \\ 6 \end{pmatrix} \right)$

e.g. coordinates of C are $(7, 7, 6)$ ***A1*** ***N1***

[3 marks]

c(i) and (ii). (i) evidence of using scalar product on $\overrightarrow{AB}$ and $\overrightarrow{AD}$ ***(M1)***

e.g. $\overrightarrow{AB} \bullet \overrightarrow{AD} = 5(1) + 2(3) + 1(2)$

$\overrightarrow{AB} \bullet \overrightarrow{AD} = 13$ ***A1*** ***N2***

(ii) $\left|\overrightarrow{AB}\right| = 5.477\ldots, \left|\overrightarrow{AD}\right| = 3.741\ldots$ ***(A1)(A1)***

evidence of using $\cos A = \frac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AD}\right|}$ ***(M1)***

correct substitution ***A1***

e.g. $\cos A = \frac{13}{20.493}$

$\widehat{A} = 0.884\ (50.6^{\circ})$ ***A1*** ***N3***

[7 marks]

d. **METHOD 1**

evidence of using area $= 2\left(\frac{1}{2}\left|\overrightarrow{AD}\right|\left|\overrightarrow{AB}\right| \sin \mathrm{D\widehat{A}B}\right)$ ***(M1)***

correct substitution ***A1***

e.g. area $= 2\left(\frac{1}{2}(3,741\ldots)(5.477\ldots) \sin 0.883\ldots\right)$

area $= 15.8$ ***A1*** ***N2***

METHOD 2

evidence of using area $= b \times h$ ***(M1)***

finding height of parallelogram ***A1***

e.g. $h = 3.741\ldots \times \sin 0.883\ldots (= 2.892\ldots)$, $h = 5.477\ldots \times \sin 0.883\ldots (= 4.234\ldots)$

area $= 15.8$ ***A1*** ***N2***

[3 marks]

Examiners report

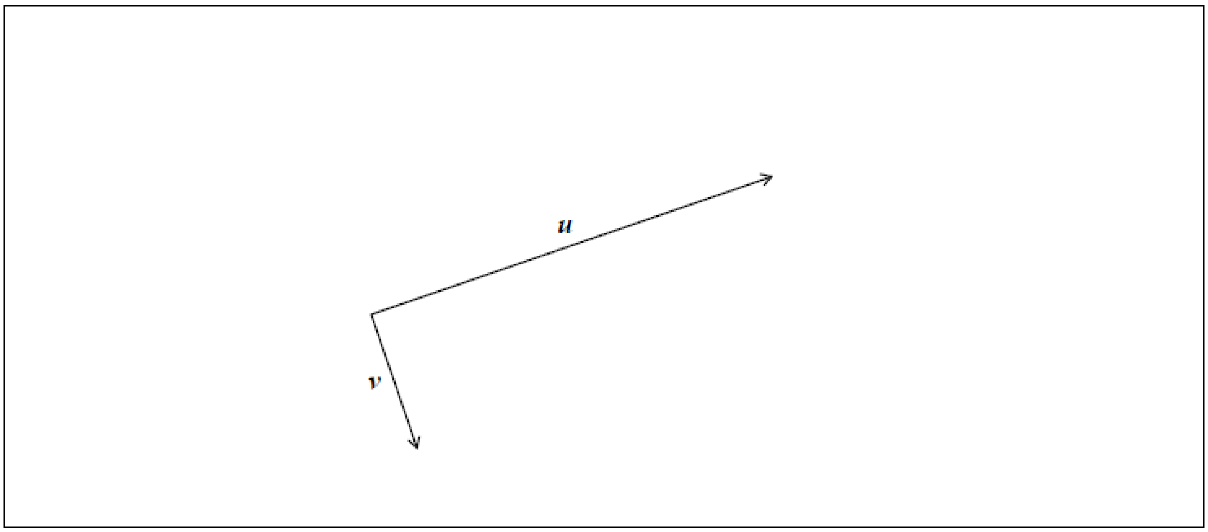
a(i), (ii) and (iii). Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

b. Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

c(i) and (ii): Some candidates were unable to find the scalar product in part (c), yet still managed to find the correct angle, able to use the formula in the information booklet without knowing that the scalar product is a part of that formula.

d. Few candidates considered that the area of the parallelogram is twice the area of a triangle, which is conveniently found using $\mathrm{B\hat{A}D}$. In an effort to find base $\times$ height , many candidates multiplied the magnitudes of $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AD}}$, missing that the height of a parallelogram is perpendicular to a base.

The following diagram shows two perpendicular vectors ***u*** and ***v***.

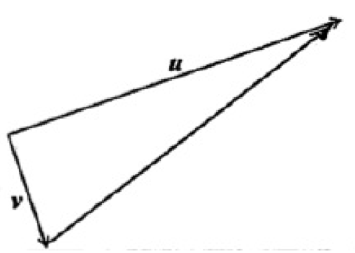


a. Let $w = u - v$. Represent w on the diagram above. [2]

b. Given that $u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find \(n\). [4]

Markscheme

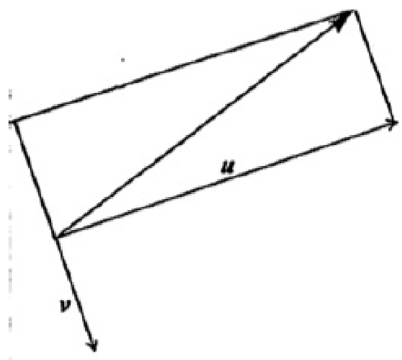
a. **METHOD 1**



A1A1 ***N2***

Note: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow).

METHOD 2



A1A1 *N2*

Notes: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow).

Additional lines not required.

[2 marks]

b. evidence of setting scalar product equal to zero (seen anywhere) ***R1***

eg $\boldsymbol{u} \cdot \boldsymbol{v} = 0,\ 15 + 2n + 3 = 0$

correct expression for scalar product ***(A1)***

eg $3 \times 5 + 2 \times n + 1 \times 3,\ 2n + 18 = 0$

attempt to solve equation ***(M1)***

eg $2n = -18$

$n = -9$ ***A1*** ***N3***

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

Consider the lines L_1 , L_2 , L_2 , and L_4 , with respective equations.

$$L_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$$L_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + p \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$L_3 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix}$$

$$L_4 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = q \begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$$

a. Write down the line that is parallel to L_4 . [1]

b. Write down the position vector of the point of intersection of L_1 and L_2 . [1]

c. Given that L_1 is perpendicular to L_3 , find the value of a . [5]

Markscheme

a. L_1 ***A1 N1***

[1 mark]

b. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ***A1 N1***

[1 mark]

c. choosing correct direction vectors ***A1A1***

e.g. $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix}$

recognizing that $\boldsymbol{a} \bullet \boldsymbol{b} = 0$ ***M1***

correct substitution ***A1***

e.g. $-3 - 4 - a = 0$

$a = -7$ ***A1 N3***

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Two points P and Q have coordinates (3, 2, 5) and (7, 4, 9) respectively.

Let $\vec{\mathrm{PR}}$ = 6***i*** – ***j*** + 3***k***.

a.i. Find $\vec{\mathrm{PQ}}$. [2]

a.ii. Find $\left|\vec{\mathrm{PQ}}\right|$. [2]

b. Find the angle between PQ and PR. [4]

c. Find the area of triangle PQR. [2]

d. Hence or otherwise find the shortest distance from R to the line through P and Q. [3]

Markscheme

a.i. valid approach ***(M1)***

eg (7, 4, 9) – (3, 2, 5) *A* – *B*

$\vec{\mathrm{PQ}} = 4\boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k} \left(= \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} \right)$ ***A1 N2***

[2 marks]

a.ii. correct substitution into magnitude formula ***(A1)***

eg $\sqrt{4^2 + 2^2 + 4^2}$

$\left|\vec{\mathrm{PQ}}\right| = 6$ ***A1 N2***

[2 marks]

b. finding scalar product and magnitudes ***(A1)(A1)***

scalar product = (4 × 6) + (2 × (−1) + (4 × 3) (= 34)

magnitude of PR = $\sqrt{36 + 1 + 9} = (6.782)$

correct substitution of **their** values to find cos $\mathrm{Q}\hat{\mathrm{P}}\mathrm{R}$ ***M1***

eg cos $\mathrm{Q}\hat{\mathrm{P}}\mathrm{R} = \frac{24-2+12}{(6)\times\left(\sqrt{46}\right)},\ 0.8355$

0.581746

$\mathrm{Q}\hat{\mathrm{P}}\mathrm{R}$ = 0.582 radians or $\mathrm{Q}\hat{\mathrm{P}}\mathrm{R}$ = 33.3° ***A1 N3***

[4 marks]

c. correct substitution ***(A1)***

eg $\frac{1}{2} \times \left|\vec{\mathrm{PQ}}\right| \times \left|\vec{\mathrm{PR}}\right| \times \sin P,\ \frac{1}{2} \times 6 \times \sqrt{46} \times \sin 0.582$

area is 11.2 (sq. units) ***A1 N2***

[2 marks]

d. recognizing shortest distance is perpendicular distance from R to line through P and Q ***(M1)***

eg sketch, height of triangle with base $[\mathrm{PQ}]$, $\frac{1}{2} \times 6 \times h,\ \sin 33.3^\circ = \frac{h}{\sqrt{46}}$

correct working ***(A1)***

eg $\frac{1}{2} \times 6 \times d = 11.2,\ \left|\vec{\mathrm{PR}}\right| \times \sin P,\ \sqrt{46} \times \sin 33.3^\circ$

3.72677

distance = 3.73 (units) ***A1 N2***

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

Markscheme

evidence of equating vectors ***(M1)***

e.g. $L_1 = L_2$

for any **two** correct equations ***A1A1***

e.g. $2 + s = 3 - t$, $5 + 2s = -3 + 3t$, $3 + 3s = 8 - 4t$

attempting to solve the equations ***(M1)***

finding **one** correct parameter $(2 = -1, t = 2)$ ***A1***

the coordinates of T are $(1, 3, 0)$ ***A1*** ***N3***

[6 marks]

Examiners report

Those candidates prepared in this topic area answered the question particularly well, often only making some calculation error when solving the resulting system of equations. Curiously, a few candidates found correct values for *s* and *t*, but when substituting back into one of the vector equations, neglected to find the *z*-coordinate of T.

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

Markscheme

appropriate approach ***(M1)***

eg $\begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, $L_1 = L_2$

any two correct equations ***A1A1***

eg $10 + 2s = 2 + 3t$, $6 - 5s = 1 + 5t$, $-1 - 2s = -3 + 2t$

attempt to solve ***(M1)***

eg substituting one equation into another

one correct parameter ***A1***

eg $s = -1$, $t = 2$

correct substitution ***(A1)***

eg $2 + 3(2)$, $1 + 5(2)$, $-3 + 2(2)$

A = (8, 11, 1) (accept column vector) ***A1*** ***N4***

[7 marks]

Examiners report

Most students were able to set up one or more equations, but few chose to use their GDCs to solve the resulting system. Algebraic errors prevented many of these candidates from obtaining the final three marks. Some candidates stopped after finding the value of s and/or t.